Physica A 389 (2010) 3870–3875

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Spontaneous symmetry breaking in asymmetric exclusion process with constrained boundaries and site sharing: A Monte Carlo study

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ARTICLE INFO

Article history:
Received 4 December 2009
Received in revised form 30 March 2010
Available online 4 June 2010

ABSTRACT

This paper investigates the two species totally asymmetric simple exclusion process (TASEP) with constrained boundaries and site sharing in a one-lane system. The model is reminiscent of pedestrian traffic crossing a narrow pathway in both directions. In boundaries, particles can enter the system only if the corresponding sites are empty. The new aspect of this study compared to previous two species TASEP models is that the oppositely moving particles do not exchange their positions each other but by sharing the same site. Monte Carlo simulations have shown that the spontaneous symmetry breaking is observed in high–low-density phase and asymmetric low–low-density phase. The flipping processes are also observed in both phases. The maximal current phase appears for sufficiently large sharing probability. Histograms of two species of particles and average currents are computed. The results are also compared with the Bridge model [Evans et al., Phys. Rev. Lett. 74 (1995) 208] which means that two species of particles can exchange their positions with a certain probability when they meet together. It is shown that our model exhibits higher current than that in the Bridge model.

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1. introduction

Non-equilibrium systems have attracted the interest of interdisciplinary researchers because a variety of interesting phenomena such as boundary-induced phase transitions, phase separations, and spontaneous symmetry breaking are observed. Originally introduced in the description of ribosome motion along mRNA in 1968 [1], totally asymmetric simple exclusion process (TASEP) and its variants have exhibited properties believed to be characteristics of many real-world non-equilibrium processes such as molecular motor traffic [2], protein synthesis [3], fungal hyphal growth [4]. On the other hand, the TASEP has also been extensively studied in its own right in the context of different particle properties (e.g., large particles [5–7], two species of particles [8,9] and different lattice geometries (e.g., multiple channels [10–12], junctions [13,14]) as well as different updating procedures (e.g., random update [15], parallel update [16]). These investigations enhanced a broader understanding of non-equilibrium systems.

Recently, the study on spontaneous symmetry breaking (SSB) in non-equilibrium systems has received much attention using TASEP with two species of particles. The SSB in this way is characterised by unequal densities of two species of particles. Evans et al. [8] firstly observed the SSB in one-dimensional two species TASEP with open boundary conditions. In their model, two species of particles can exchange their positions with a certain probability when they meet together. As the shape of the model in Ref. [8] looks like a bridge, the model is known as the "Bridge model". In the Bridge model, it was shown that

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doi:10.1016/j.physa.2010.05.028

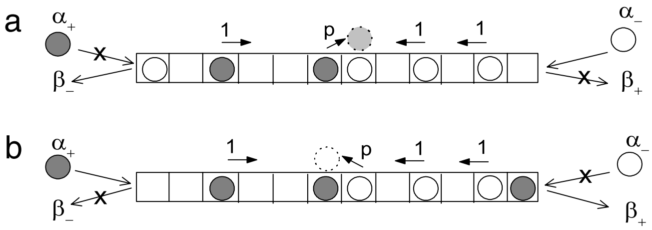

Fig. 1. Diagrammatical representation of a one-dimensional TASEP with two species of particles. The (+) particles move from the left to the right, represented by filled circles, while the (−) particles (denoted by open circles) move from the right to the left. Arrows mean the possible movements. Symbols over the arrows indicate the corresponding hopping probabilities. A site can be shared with probability q by two species of particles when they meet each other. (a): prohibited entrance for (+) particles, (b): prohibited entrance for (−) particles.

a high-density–low-density (HD/LD) phase and an asymmetric LD/LD phase could exist and both of them exhibit broken symmetry. Levine and Willmann [17] extended the Bridge model by considering Langmuir Kinetics (LK) on a lattice. Two species of particles are assumed to have the same attachment rate and detachment rate. They found that the SSB could exist and the localized shocks appear in some conditions. The SSB has also been investigated in multiple-channel TASEPs with random update [18] and parallel update [19,20]. Popkov et al. introduced the Bridge model fed by two junctions [21]. The SSB is observed as well. In addition, a co-existence region between the symmetry-broken phase and the low-density symmetric phase exists in their system. More recently, Gupta et al. [22] generalized the Bridge model by considering the boundary exchange of two species of particles with probability γ. Their results confirm the existence of SSB for non-zero exit rate β and γ, provided both value are not too large. Their investigation demonstrates the robustness of the SSB in the modified Bridge model [22]. Most of the modified Bridge models are based on the special case of exchanging probability $q = 1$. Theoretical results for general q $(0 \leq q \leq 1)$ have not been obtained in those models although the phenomenon of the SSB persists qualitatively for $q \neq 1$ [22].

One of the remaining intriguing open questions related to the Bridge model is whether the asymmetric LD/LD phase exists or not. Arndt et al. [23] have argued that the asymmetric LD/LD phase does not exist based on the numerical studies. Interestingly, simulation results reported by Clincy et al. [24] have shown that there exists two unequal LD phases. However, they do not correspond to the predicted asymmetric LD/LD phase. Erickson et al. [25] also revisited the Bridge model via high-precision Monte Carlo data and associated their work with the study of traffic on a narrow bridge. Their simulation results show that the LD/LD phase will disappear if the system size is sufficiently large and/or the exchange probability is sufficiently low.

In these TASEP models, either one species or two species, particles follow the hard-core exclusion, that is, each site can be occupied by at most one particle at the same time. This paper investigates a one-dimensional TASEP model in which two species of particles move along a single lane in opposite directions. The same species of particles still follow hard-core exclusion, but different species particles may share a site at probability q $(0 \leq q \leq 1)$ which is the main difference between our model and previous TASEP models. For simplicity, we call the model the *site-sharing* model. This model may be relevant for pedestrian traffic. When pedestrians walk along a single-channel pathway in opposite directions and meet together, they may share a site, and then pass each other.

We studied the site-sharing model using extensive Monte Carlo simulations since theoretical analysis for the site-sharing model with general q $(0 \leq q \leq 1)$ has not been conducted so far. The spontaneous symmetry breaking is observed. Our work further supports the conclusion that the SSB is robust with respect to changes in the microscopic dynamics of the site-exchanging and site-sharing models. Phase diagram, bulk density and particle currents are computed. For comparison, we also examine the Bridge model. It is shown that our model exhibits higher current than the Bridge model in the high-density phase. This paper is organized as follows. In Section 2, the model is formed, followed by simulation results in Section 3. We give our conclusions in Section 4.

2. Model description

An illustration of a one-dimensional TASEP with two species of particles is shown in Fig. 1. The system size is assumed to be N. Each site can be occupied by a (+) particle and/or a (−) particle, or empty. The (+) particles move from the left to the right, represented by filled circles, while the (−) particles denoted by open circles move oppositely (see Fig. 1). The model is symmetric with regard to the rules and two species of particles. Therefore, we here define the rules of (+) particles. The (−) particles perform the similar rules from the right to the left. For simplicity, we assume $\alpha_+ = \alpha_- = \alpha$ and $\beta_+ = \beta_- = \beta$ in simulations. In each time step, a site i is randomly chosen. A probability for choosing a (+) or (−) particle at site i is equal, i.e., 0.5.

- When i is in the bulk $(1 < i < N)$,
  1. A (+) particle at site i can hop to site $i + 1$ with probability 1 if the target site is empty;
  2. If the target site is occupied by a (−) particle, the (+) particle can share the site with probability q $(0 \leq q \leq 1)$;
  3. If the target site is occupied by the same species particle, the (+) particle stays at site i.

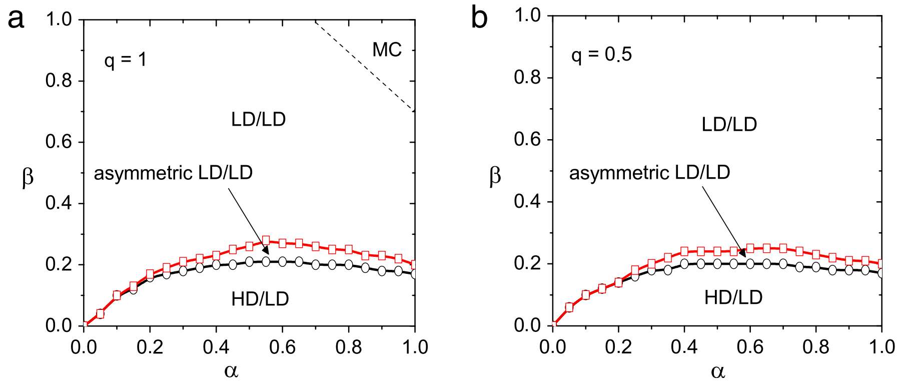

Fig. 2. (Color online) Phase diagram of the TASEP with two species of particles and site sharing for different sharing probabilities q. The red open squares correspond to the boundary between the symmetric LD/LD and asymmetric LD/LD phases, while the black open circles denote the boundary between the asymmetric LD/LD and HD/LD phases. The lines are guides for eyes. (a) $q = 1$ and (b) $q = 0.5$.

- When i is in the boundaries,
  1. $i = 1$. A $(+)$ particles can enter the left boundary with rate α_+ only if the first site is empty. If the site is occupied by the other $(+)$ particle, the $(+)$ particle already at site 1 can hop to site 2 with probability 1 if site 2 is empty or with probability q if site 2 is occupied by a $(-)$ particle;
  2. $i = N$. A $(+)$ particle can exit the system from the last site with rate β_+.

3. Simulation results and discussion

To investigate the dynamics of the system, Monte Carlo simulations are carried out. Open boundary conditions and random update are used with the system size $N = 1000$. The first 1×10^9 time steps are discarded to let the transient out. The phase diagram, stationary current and density profiles are obtained by averaging 2×10^9 time steps. The phase diagram is simulated for $q = 0.5$, 1 and shown in Fig. 2. The red open squares correspond to the boundary between the LD/LD and asymmetric LD/LD phases, while the black open circles denote the boundary between the asymmetric LD/LD and HD/LD phases. When $q = 1$, a $(+)$ particle does not distinguish between a $(-)$ particle and a hole. And similarly for a $(-)$ particle. In this case, four stationary phases exist in the system, that is, symmetric LD/LD, asymmetric LD/LD, HD/LD and MC phases (see Fig. 2(a)). We note that the MC phase covers a small region in the site-sharing model, while it is reduced to a point ($\alpha = 1, \beta = 1$) in the Bridge model. Our simulation results show that the MC phase will disappear when q is approximately $q < 0.97$. In other words, there are only three phases in the system for $q < 0.97$ (see Fig. 2(b)). The reason for the disappearance of the MC phase is probably that the site-sharing probability q is not too large which limits the interaction between two species of particles. However, this explanation requires support from theoretical analysis. Furthermore, when q decreases approximately to $q \leq 0.3$, the asymmetric LD/LD phase will almost reduce to a curve rather than a region. In such conditions, there is only one symmetry-breaking transition from the LD to the HD/LD in this model.

We then investigate histograms $P(\rho_+, \rho_-)$ of particle densities, where ρ_+ and ρ_- are instantaneous densities of $(+)$ and $(-)$ particles, respectively. Fig. 3 shows four typical particle density histograms in the HD/LD, asymmetric LD/LD, LD/LD and MC phases, respectively. One can see that in the HD/LD phase, a double peak with two off-diagonal maxima appears, while in the symmetric LD/LD and MC phases, a single peak exists on the diagonal.

The flipping process is shown in Fig. 4. The density difference $\rho_+ - \rho_-$ has been measured as functions of time. The flipping processes of the HD/LD and asymmetric LD/LD phases are observed clearly in Fig. 4(a) and (b). The system flips between positive net values and negative net values. The positive (negative) net values imply that the bulk density of positive (negative) particles are larger than that of negative (positive) particles. This means the existence of the SSB in the system.

We performed computer simulations with different system length (up to $L = 10{,}000$) to study the finite-size effect in our model (see Fig. 5). It is shown that the phase boundary between the asymmetric LD/LD and symmetric LD phases little depend on the system size, while the region of the asymmetric LD/LD phase seems to shrink and then keep unchanged with the increase of the system size. This suggests that the asymmetric LD/LD phase probably exists in the thermodynamic limit ($L \to \infty$).

Stationary currents in the present model are investigated. Due to the flipping phenomenon in the model, we calculate the average current of $(+)$ and $(-)$ particles as the system current, i.e., $J_{ave} = (J_+ + J_-)/2$, where J_+ and J_- are currents of $(+)$ and $(-)$ particles, respectively. For simplicity, we assume that $q = 0.5, 1$, $\beta = 0.3, 0.6, 0.9$ while α changes from 0 to 1. Fig. 6(a) shows the stationary current obtained from computer simulations for $q = 1$. With the increase of β, the average current increases as well. However, when $q \neq 1$ (e.g., $q = 0.5$), an unexpected phenomenon appears. The average current first increases upon increasing β, and then reaches the maximal current (see Fig. 6(b)). In other words, J_{ave} is maintained and its value is dictated by q rather than α or β even the system is in the symmetric LD phase. We also observe that the maximal current region shrinks with the increase of q.

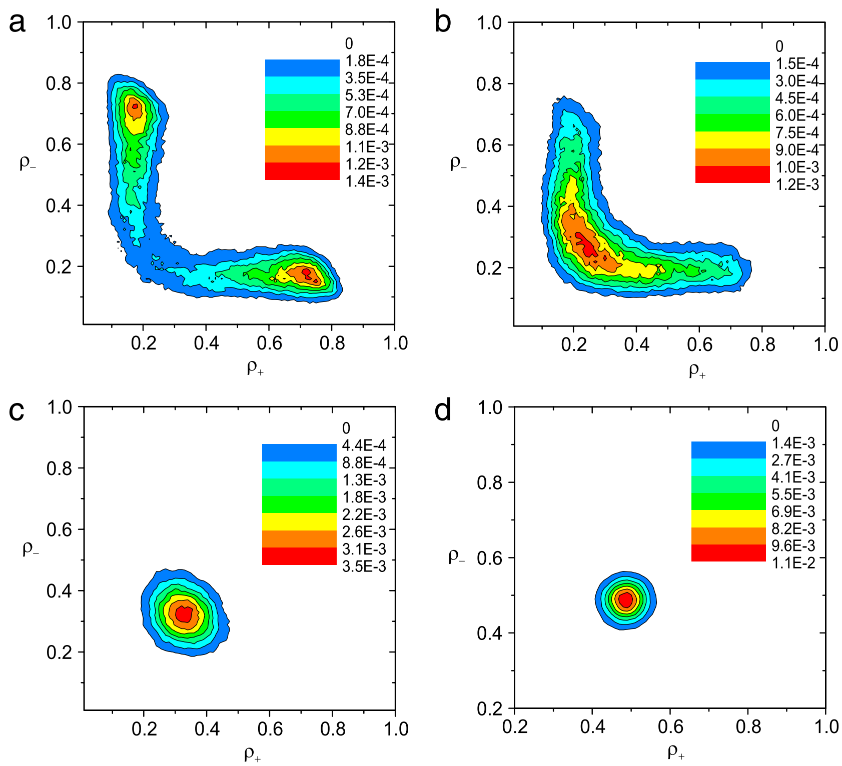

Fig. 3. The simulation results of densities with (a) HD/LD phase: $\alpha = 0.8$, $\beta = 0.16$ and $q = 0.8$; (b) Asymmetric LD/LD phase: $\alpha = 0.8$, $\beta = 0.26$ and $q = 0.8$; (c) LD phase: $\alpha = 0.8$, $\beta = 0.4$ and $q = 0.8$; (d) MC phase: $\alpha = 1$, $\beta = 1$ and $q = 1$.

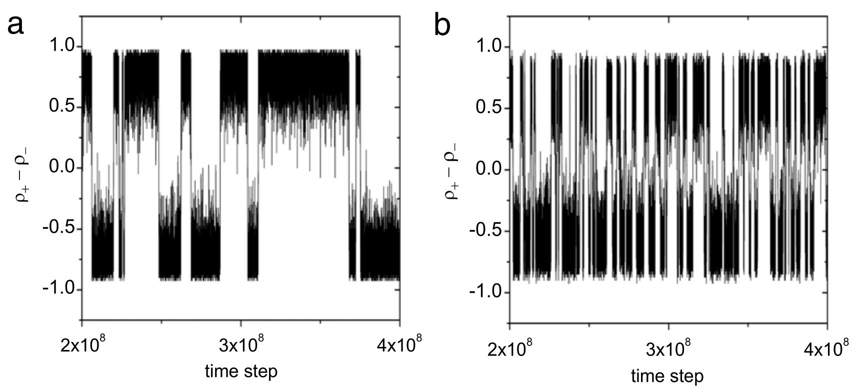

Fig. 4. Illustration of flipping processes of spontaneous densities in two breaking phases with $q = 0.5$ and $N = 40$. (a) HD/LD phase: $\alpha = 0.4$ and $\beta = 0.1$; (b) Asymmetric LD/LD phase: $\alpha = 0.4$ and $\beta = 0.16$.

We finally compared the average current between our model and the Bridge model under the same α, β and q. We assume that $\alpha = 0.4, 1$, $q = 0.3, 0.6, 0.9$ while β changes from 0 to 1 so that we can observe the current in all possible phases. It is shown that our model can lead to a higher current than that in the Bridge model (see Fig. 7(a)). The reason for this is due to the site-sharing mechanism in our model rather than the site-exchanging mechanism in the Bridge model.

4. Conclusion

The totally asymmetric simple exclusion process (TASEP) with two species of particles in a one-lane system is studied. The model is reminiscent of pedestrian traffic crossing a narrow pathway in both directions. Two species of particles move oppositely and can enter the system only if the corresponding sites are empty. Hard-core exclusion is applied to the same species of particles while different species of particles are allowed to share the same site at a certain probability q. This kind of sharing effect has not been investigated in previous TASEP models, to the best of our knowledge. There are four possible

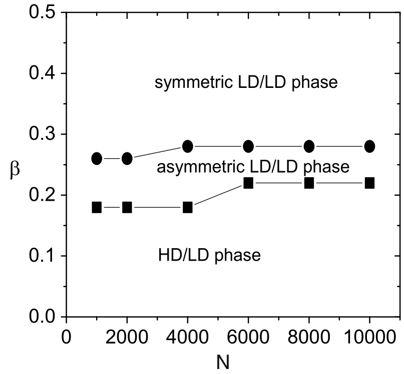

Fig. 5. The size effect with $q = 1$, $\alpha = 0.6$ and different system sizes.

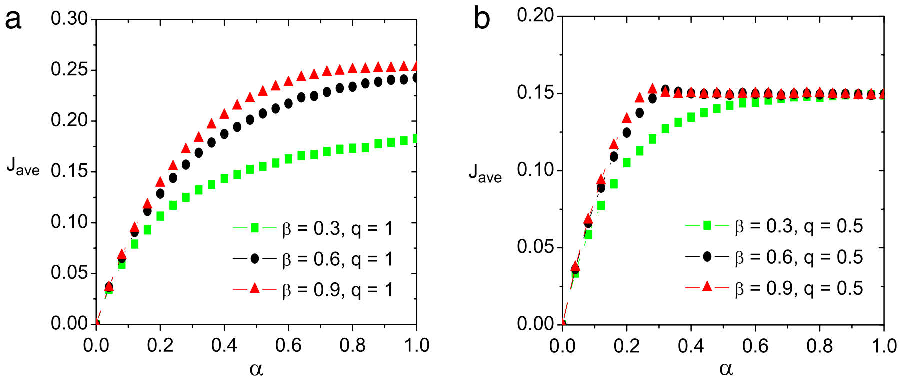

Fig. 6. (Color online) The stationary current with different β. J_{ave} is the average current of $(+)$ and $(-)$ particles, i.e., $J_{ave} = (J_+ + J_-)/2$. (a) $q = 1$ and (b) $q = 0.5$.

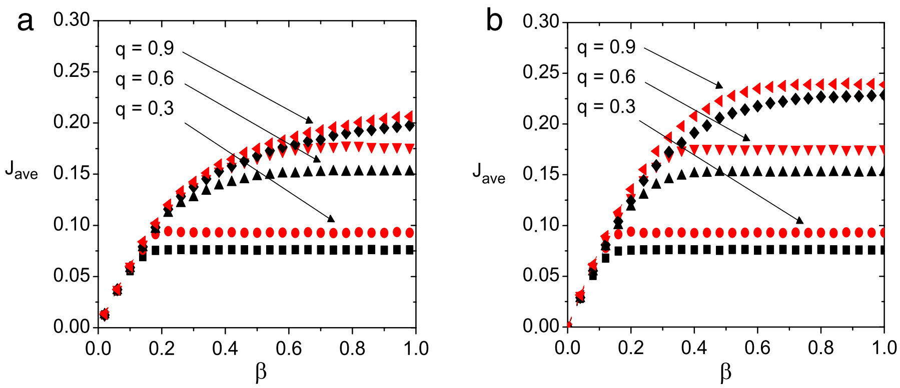

Fig. 7. (Color online) The stationary current with different q. The red symbols correspond to our model, while the black symbols are for the Bridge model. (a) $\alpha = 0.4$ and (b) $\alpha = 1$.

phases in the system, i.e., MC, symmetric LD/LD, asymmetric LD/LD and HD/LD. The spontaneous symmetry breaking (SSB) is observed in the two phases: HD/LD and asymmetric LD/LD. With the decrease of q, the asymmetric phase reduces to the boundary between the symmetric LD/LD and the HD/LD phases. The MC phase will disappear when $q < 0.97$. The histograms of two species of particles and the flipping process are plotted. Our model exhibits higher current, compared to the Bridge model, which is due to the site-sharing mechanism in our model. More interestingly, it is shown that the average current in the symmetric LD/LD phase is determined by q rather than α or β when $q \neq 1$.

Our work shows that the sharing effect on the TASEP is an interesting topic and needs to be further investigated. The present model has been investigated using extensive Monte Carlo simulations. However, it has not been conducted by theoretical analysis. It is of interest to revisit the problem of the spontaneous symmetry breaking in the site-exchanging or site-sharing models with general q $(0 \leq q \leq 1)$.

Acknowledgements

The authors are grateful to the anonymous referees for their valuable comments and suggestions.

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